

Open Inflation*

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Open Inflation has recently been suggested as a possible way out of the age crisis caused by observations of a large rate of expansion of the universe, in conflict with the existence of very old globular clusters. It proposes that our local patch of the universe originated in a quantum tunneling event, with the formation of a single bubble within which our universe inflated to almost flatness. I review the different models proposed together with their predictions for the amplitude of temperature anisotropies in the cosmic microwave background.

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I. INTRODUCTION

Until recently, one of the most robust predictions of inflation was the extreme flatness of our local patch of the universe. However, in the last few months there has been a lot of excitement about the possibility of producing an open universe from inflation [1–3]. An open universe could resolve the age crisis caused by the observations of a relatively large Hubble constant, $H_0 = 69 \pm 8$ km/s/Mpc, which corresponds (for $\Omega_0 = 1$ and $\Lambda = 0$) to a very small age of the universe, $t_0 = 9.5 \pm 1.1$ Gyr [4], in conflict with the age of globular clusters, 12–17 Gyr [5]. An alternative solution could be the introduction of a non-zero cosmological constant Λ which could accommodate both a flat and old universe with a large expansion rate, but there still remains the question of why Λ is so small. With ‘open inflation’ it is possible to produce a very homogeneous open universe, without the need to introduce a cosmological constant.*

In standard inflation the homogeneity and flatness problems of the hot big bang cosmology are intimately related and it is not possible to relax one (flatness) without affecting the other (homogeneity) [6]. During inflation both curvature and inhomogeneities are stretched away. The present value of $\Omega = \rho/\rho_c$ depends on the number of e -folds N_e during inflation, $|1 - \Omega_0| \simeq 10^{57} e^{-2N_e}$, and thus a large expansion, $N_e \gg 65$, produces an exponentially flat universe, with the size of the homogeneous patch L_0 much greater than the present horizon, H_0^{-1} . In order to produce an open universe, $\Omega_0 \lesssim 1$, one requires $N_e \lesssim 65$, which means that $L_0 \simeq H_0^{-1}$. On the other hand, assuming that the density perturbations at large scales $L \geq L_0$, with density contrast $\delta_L \simeq 1$, are a typical realization of a homogeneous Gaussian random field, Grishchuk and Zel’dovich showed [7] that they contribute to the amplitude of the quadrupole anisotropy of the cos-

mic microwave background (CMB) as $Q \simeq (LH_0)^{-2}$. By constraining $Q_{\text{rms}} \lesssim 2 \times 10^{-5}$ from COBE [8], one can see that the size of the homogeneous patch should be at least $L_0 \gtrsim 500H_0^{-1}$ [6]. This requires $N_e \gtrsim 70$ or $|1 - \Omega_0| \lesssim 10^{-4}$. Thus in standard inflation, an open universe with Ω_0 significantly less than one is incompatible with the large scale homogeneity we observe in the CMB.

II. MODELS OF OPEN INFLATION

Open inflation solves the homogeneity problem by inflating the universe in a false vacuum and then creating a bubble within which our patch of the universe expanded to ‘almost’ flatness, thanks to the energy density of an inflaton field. According to this picture we live inside a bubble that nucleated from de Sitter space by quantum tunneling with an extremely small probability. This ensures two things, first that there will be no collisions with other bubbles, at least in our past light cone, and second that the nucleated bubble is extremely spherically symmetric, defining a very homogeneous initial hypersurface. The bubble walls then expand at the speed of light, while the space-time within the bubble becomes that of an open universe [9], with infinite equal-time hypersurfaces.

The first models of open inflation [1,2] considered a single scalar field trapped in a metastable state that later tunneled to the true vacuum with a non-zero energy density. The field then rolled down a very flat potential, inflating the required amount of e -folds to give an open universe. Note that in order to produce a universe with say $\Omega_0 = 0.3$, we have to fine-tune the number of e -folds to within one percent, which is a rather mild fine-tuning. These models, though, had the unpleasant feature of strongly contrived potentials for the inflaton field. In order to tunnel without producing too large inhomogeneities on large scales, we need a large mass in the false vacuum. One of the dangers of quantum tunneling with a small mass is the existence of the Hawking-Moss instanton [10]. In this case, the field jumps to the top of the barrier between the two vacua and very slowly ‘rolls

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down' the potential. Very large amplitude quantum fluctuations are then produced that are not inflated away and would unacceptably distort the observed anisotropy of the CMB. For that reason alone it is assumed that the mass of the tunneling field should be much larger than the rate of expansion at the false vacuum. On the other hand, a very small mass of the inflaton field is required after tunneling to give the observed amplitude of density perturbations in the CMB. Linde and Mezhlumian [3] suggested a simple way out by including two fields, one with a large mass, responsible for tunneling, and the other with a very small mass, responsible for inflation inside the bubble. Except for the finite number of e -folds, the second phase of open inflation is identical to standard inflation. The scalar field ends inflation by oscillating at the bottom of the potential and releasing its potential energy density into radiation. The standard hot big bang cosmology follows thereafter.

III. DENSITY PERTURBATIONS

If open inflation is to be a good model of the universe it should not only solve the flatness and homogeneity problems but also account for the temperature fluctuations we observe in the CMB. There are in principle two sources of curvature perturbations in open inflation, the quantum fluctuations of the inflaton field that are stretched to cosmological scales by the expansion, and quantum fluctuations in the bubble wall produced during the bubble nucleation. The former have been thoroughly studied in recent papers [1,2,11]. Their main result is the existence, in the spectrum of a scalar field with $m^2 < 2H^2$ in the false vacuum, of a single discrete supercurvature mode that has to be taken into account together with the subcurvature modes. In single field open inflation [1,2], we have seen that such a small mass is incompatible with observations, but in two-field models [3] the inflaton field could indeed have a very small mass in the false vacuum. This means that the universe could actually be in a process of self-reproduction, and thus extremely inhomogeneous [12]. In that case, very large scale density perturbations could affect the amplitude of the lower multipoles of the temperature anisotropies, as discussed in the introduction. This is the so-called Grishchuk-Zel'dovich effect [7]. We have recently evaluated this effect in the open universe case [13] and found strong constraints on the amplitude of very long wavelength perturbations contributing to the first CMB multipoles. These constraints can be used [14] to bound the mass of the inflaton field in the false vacuum.

But there are also potentially dangerous curvature perturbations arising from quantum fluctuations of the bubble wall at the moment of tunneling. They have been addressed in a very qualitative way by Linde and Mezhlumian [3], and later studied in detail in Ref. [14]. Most of the results of Refs. [1–3] were done in the thin wall

approximation, which is valid for most potentials with a deep false vacuum minimum and a large potential barrier between the two vacua. They also assume the tunneling occurs from de Sitter to Minkowski space-time. However, the new ingredient in open inflation is precisely the non-zero energy density of the true vacuum which could still drive inflation to almost flatness. The instanton action associated with the more general quantum tunneling process from de Sitter to de Sitter was computed long ago by Parke [15]. It is possible to calculate the tunneling action of open inflation, and then compute the amplitude of curvature perturbations from quantum fluctuations in the bubble wall, following the covariant formalism of Ref. [16].

The origin of curvature perturbations in the bubble wall can be understood as follows. Due to a large instanton action, the probability of tunneling $\Gamma \sim \exp(-S_E)$ is extremely small. The radius of curvature of the bubble corresponds to an extremum of the instanton action, and the amplitude of quantum fluctuations in this radius can be obtained from the second derivative of the instanton action. These fluctuations can be understood as long wavelength homogeneous perturbations in the curvature of the bubble wall. In the open de Sitter coordinates, the bubble wall is a time-like hypersurface at a fixed radial coordinate σ which asymptotically determines a space-like hypersurface at a fixed comoving time η inside the bubble. Thus perturbations in the radius of the bubble propagate inside as perturbations in the time it takes to end inflation [14]. This generates curvature perturbations inside the bubble, very much like adiabatic density perturbations from quantum fluctuations of the inflaton field [17]. One can show [14] that for most open inflation models, due to the gravitational effects at bubble nucleation, the amplitude of bubble wall fluctuations can be much smaller than those of the inflaton field in the subsequent phase of inflation inside the bubble.

The present models of open inflation seem to work quite well with very reasonable parameters, at least as reasonable as those of standard inflation. A different issue is whether these models will turn out to be the correct description of the origin of our patch of the universe. Fortunately, cosmology has become a science and within a few years we will be able to tell, from the shape and amplitude of the spectrum of density perturbations in the cosmic microwave background, whether our patch of the universe is indeed open or flat [18]. In any case, it is encouraging to see that the inflationary paradigm is able to accommodate an open universe, even if we never have to make use of it.

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